The Elastic Stability of Palms
Peter Sterken

Abstract:
During pulling tests on palms, the moduli of elasticity were measured along the stems of several palms (Phoenix sp. and Washingtonia sp.). Both breaking and uprooting were not possible to predict using inclinometers and elastometers. These breaking experiments on palms have been the basis of this paper and its proposals. A mathematical model is presented with which visual palm assessment can be enhanced. The guidelines given combine the visual assessment of mechanical catastrophic behaviours with the safety factor regarding elastic stability and the wind load analysis for cabling the palm tree. Furthermore, one of the most frequent preoccupations of climbers is the possible failure of the trunk of slender palms, caused partially by the added weight of the climber. With the Greenhill equation, the difference between e.g. a thick Phoenix canariensis and more slender palm species, in terms of their safety, can be quantified. The slenderness, elasticity and weight of the palm, together with the eventual weight of the climber, are amongst the vital factors that govern its elastic stability.

A new addition is the theory of elastic stability. Earlier components of this model have recently been published in the scientific peer-reviewed Arboricultural Journal, Vol. 29, pp 243-265.

Key-words: Theory of elastic stability · Palms · Safety · Critical wind speed

Published in:


Koninklijke Belgische Bibliotheek. 2008. BRO 2008 1.474

Cited by:


© Peter Sterken, 2008
Introduction
A mathematical model is presented which goal is to enhance visual palm assessment.

Firstly, the safety factor of the palm trunk regarding elastic stability is calculated. This factor has to be higher than 100%, in order not to buckle under its own weight. If the palm’s elastic stability is maintained, the palm can withstand a certain amount of additional loads, like the weight of a climber or wind loads.

Secondly, the additional wind loads are estimated which enables to optimize artificial supports of the palm. Cabling the assessed palm to other palms or structures is a very efficient solution for reducing oscillations caused by vortex shedding and turbulent gusts. This has also proved to be efficient for diminishing the risk for uprooting or breakage in palms that have a limited safety factor. The wind load in the palm, and the resulting loading of the supporting structure, has to be assessed undeniably and can be estimated with this model. The input of the expected wind speed for the area, temperature and altitude, enable to optimise this wind load analysis.

Thirdly, a hypothesis has been formulated which could heighten the efficiency of visual palm assessment. It is suggested that the critical wind speed for failure of the palm stem depends significantly on the relationship between the modulus of elasticity, the form of the cross-section, the slenderness of the palm, its elastic stability, dynamic wind loading and mechanical behaviours. These mechanical behaviours, e.g. shear, splitting and Brazier buckling, can cause failure of the stem while these seem not predictable by means of mathematical calculations or instrumental methods.

A new addition is the theory of elastic stability. Theories of Euler, Bernoulli and Greenhill are introduced. An important finding regarding the failed *Washingtonia robusta* palms of the Atocha train station of Madrid will be commented briefly.

As far the author’s knowledge reaches, the only guideline that can be given until now is to combine the visual assessment of mechanical catastrophic behaviours with the wind load analysis for cabling the palm tree.

1. Theory of elastic stability

- *The Euler - Bernoulli theory*

  The Euler - Bernoulli theory can be employed for calculating the deflection of the palm stem, assuming it behaves according to linear isotropic elasticity:

  \[
  \frac{\partial^2 u_x}{(x^2) = \frac{M_y(x)}{EI_y}
  \]

  Where:

  - \(u_x\) = the deflection perpendicular to the beam;
  - \(x\) = the location on the beam axis;
  - \(M_y(x)\) = the internal bending moment as a function of \(x\);
  - \(E\) = the modulus of elasticity of the material;
  - \(I_y\) = the moment of inertia of the cross-section of the beam.

  The critical limit for deformation could be calculated with this theory, if the palm stem behaved as beam made of isotropic material experiencing no inner shear and being the compression strength of the wood known. Nevertheless, it is still not acknowledged if palm stems behave as such. Furthermore, it is known that the stability of plant stems cannot always be predicted in this way. Many times, they can fail – buckle - even before the compression strength of the wood is reached. Hence, in the following lines the *theory of elastic stability* is introduced, by which the safety factor of the palm is estimated.

- *The Greenhill equation*

  When the trunk is considered as a uniform column free of structural defects, its critical height, which if trespassed would cause elastic instability of the trunk, can be estimated. Greenhill’s equation can be employed if the weight of the trunk is significant (> 30%) compared to an axial
compressive weight on the trunk (e.g. the crown, fruits and climber’s weight). Greenhill’s (1881) equation is the basis for calculating the safety factor of the palm stem against buckling:

\[
H_{\text{crit}} = C \left( \frac{E}{\rho_{\text{material}}} \right)^{1/3} d^{2/3}
\]

Where:
- \( H_{\text{crit}} \) = the critical buckling height;
- \( C \) = the constant of proportionality;
- \( E \) = the modulus of elasticity of the material;
- \( \rho_{\text{material}} \) = the specific weight of the material;
- \( d \) = the diameter of the stem.

One of the most frequent preoccupations of climbers is the possible failure of the trunk of slender palms, caused partially by the added weight of the climber. One way to assess this problem would be to employ the critical weight calculation (photograph 1 and Fig. 1), according to Timoshenko & Gere (1961) and Schulgasser & Witztum (1997). In this way, the safety of the climber could partially be assessed if the stem behaved as such:

\[
W_{\text{crit}} = \beta \frac{EI}{h^2}
\]

Where:
- \( W_{\text{crit}} \) = the total critical weight that would cause buckling of the trunk;
- \( \beta \) = the constant that expresses the proportion between the axial compressive weight on the free end of the trunk and the weight of the trunk itself;
- \( h \) = the height of the trunk.

**Photograph 1 and figure 1:** the buckling problem schematically represented.

It should be noted that the capacity of palms to make their tissues stiffer and denser (see the requirements of the above equation) as their height and age heightens, is not unlimited. Therefore, their weight, growth in height and stability isn’t either. An example is the failure of several slender *Washingtonia robusta* palms of the Atocha train station of Madrid, Spain. This case suggested that these palms initially became elastically unstable under their own weight, by exceeding their critical stem height. The weight of the crown, the horizontal displacement and
resulting gravity forces would then further influence the failure process. This model could predict this failure, provided the theory of linear isotropic elasticity were appropriate. The only remaining palm tree (photograph 2) would, at the time of the assessment (12-6-2003)*, also be at its structural limit according to its diameter, density of the wood and the assumed modulus of elasticity. The latter is the average $E$ as measured through pulling tests in a standing *Washingtonia* in Terrassa, Spain, by the author. (*Note: in May 2016, the palm had already ceased to exist*)

As the reader might note, non-uniform mechanical stem properties seem here an aspect to be accounted for. It is known, that both $E$ and $\rho_{\text{material}}$ vary among palm species, among specimens of the same species and over the cross-section and height of one single trunk (Rich, 1987). However, Niklas (1994) suggested afterwards a comparatively constant $\left(\frac{E}{\rho_{\text{material}}^3}\right)^{1/3}$ value among diverse palm species. This value, analogous to the experimental results in the *Washingtonia*, has been employed in the present model. Values for *Cocos nucifera* seem to be similar as well (Ling-Long Kuo-Huang et al., 2004). Other publications report slightly higher values for coconut and oil palm trees. Nonetheless, employing the lower value involves a lower calculated limit for $H_{\text{crit}}$ and hence a wider margin for errors.

Whenever this value and $d$ are known, it is assumed that the equation predicts with precision the beam its safety. During pulling tests on palms, the moduli of elasticity were recorded. However, breaking and uprooting was not possible to predict using inclinometers and elastometers.

**Photograph 2:** The remaining palm in the Atocha train station. Current calculations indicate its limited safety margin (163.83%). Although this result (>100%) might explain why it is still standing, to this calculation the weight of the wet crown (due to the watering from the station’s ceiling) and possible inclination should also be summed. Its final safety diminishes then more towards the minimum 100%, thus not satisfying any required, in engineering terms, safety margin. The solution was to cable this palm to the station’s ceiling. The two failed palms, with similar dimensions, probably reached or surpassed their critical buckling height.
Figure 2: Breaking experiment on a damaged Phoenix dactylifera and maximum breaking load. Photograph 3: measuring the elastic modulus MOE prior to breaking.

Considerations
The present protocol is based on the following assumptions (Niklas, 1994):

The mechanical behaviour of the trunk is governed by the mechanical and physical properties of a single tissue which provides the principal stiffness against bending;

The proportion between the modulus of elasticity and the density of the tissue is constant, even among diverse palm species;

The equations predict adequately $H_{\text{crit}}$ of a beam whenever $\left( \frac{E}{\rho} \right)^{1/3}$ and $d$ are known.

Furthermore:

The stability of the palm stem might also be governed by structural defects and diverse mechanical behaviours, like shear, stress-concentrations, fibre delamination and Brazier buckling (Sterken, 2006). The current state of knowledge on palms does not permit yet to predict precisely their influence. Hence, their visual and mathematical assessment should be incorporated in future stability assessment methods.

The constant $\beta$ should vary according to the proportion of the top weight (e.g. climber and equipment) and the weight of the palm. This is especially true for very slender palms with relatively little weight, e.g. Trachycarpus fortunei.

If the palm's elastic stability is satisfactory, the palm can withstand a certain amount of additional loads, like wind loads.

2. Wind load analysis
The mathematical structure of the complete wind load analysis, the V-model, has been extensively described in the scientific paper “Prognosis of the development of decay and the fracture safety of hollow trees” (Arboricultural Journal, Vol. 29, pp 243-265). Interested readers should consult this work. One of the basic equations is offered here below. References for the combination of cabling and wind load analysis can be found in Sterken (2005).

- Wind load in the palm
The static wind loading in the palm is calculated according to the Eurocode1 (AENOR 1998):

$$ F_{\text{palm}} = 0.5C_w \left( \frac{T}{10} \left( \frac{z^2}{z_0} \right) \rho_o \right) A v^2 $$
Where:

\[ F_{\text{palm}} = \text{the force that a gust exerts in the palm crown;} \]
\[ C_w = \text{the aerodynamic coefficient describes the flexibility that the palm employs in order to diminish the force of the wind;} \]
\[ T_o = \text{the reference temperature;} \]
\[ T = \text{the expected air temperature;} \]
\[ z = \text{the altitude at which the tree stands;} \]
\[ z_o = \text{the reference altitude;} \]
\[ \rho_o = \text{the air density at 15ºC and at sea level;} \]
\[ A = \text{the wind exposed area of the palm, calculated in accordance with AENOR (1998);} \]
\[ v_z = \text{wind speed } v \text{ at a certain height } z \text{ above ground level.} \]

The safety of palms stands partially in function of the temperature and altitude, since the density of the air influences directly the force that hurricanes exert in the palm. The equation represents the air density as a function of temperature and altitude. The following example evidences how the safety is related partially to the temperature \( T \) and to the altitude \( z \) at which the palm grows (Table 1). The higher air density results in a higher wind load and hence a decrease in safety.

Table 1. An example of the influence of the temperature \( T \) and altitude \( z \) on the wind load \( F \) in a Phoenix dactylifera, where all the other parameters were kept the same:

<table>
<thead>
<tr>
<th>Temperature: 35ºC</th>
<th>Altitude: 900m</th>
<th>Wind load: 1,82kN: 186,05 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature: -5ºC</td>
<td>Altitude: 0m</td>
<td>Wind load: 2,35kN: 239,89 kg</td>
</tr>
</tbody>
</table>

3. **Visual palm assessment**

A hypothesis on stem failure of palms has been formulated here, based on the observation of broken palms and controlled breakage experiments: the bending of the palm stem is assumed to be proportional to the wind force in the crown and inversely proportional to the stiffness of the stem. Besides, it is suggested that the critical wind speed for failure of the palm stem depends significantly on the relationship between the modulus of elasticity, the form of the cross-section, the stem's elastic stability, the slenderness of the palm, dynamic wind loading and mechanical behaviours. These mechanical behaviours, e.g. shear, splitting and Brazier buckling, can cause failure of the stem while these seem not predictable by means of current methods. Therefore, they have still to be assessed visually. A profound knowledge on the criteria that influence these catastrophic mechanical behaviours is required if the stability is to be assessed.

4. **Conclusion**

With the Greenhill equation, the difference that the climber experiences between e.g. a thick Phoenix canariensis and more slender palm species, in terms of their safety, can partially be quantified. The slenderness, elasticity and weight of the palm, together with the weight of the climber, are amongst the vital factors that govern its elastic stability. The safety factor of the palm trunk against elastic instability is calculated. Theoretically, this factor has to be higher than 100%, in order not to buckle under its own weight. If this factor is satisfied, the palm can withstand a certain amount of additional loads (if, like the weight of a climber or wind loads. The additional wind loads are estimated which enables to optimize artificial supports of the palm. Cabling the assessed palm to other palms or structures is a very efficient solution for reducing oscillations caused by vortex shedding and turbulent gusts. This has also proved to be efficient for diminishing the risk for uprooting or breakage. In this case the wind load in the palm, and the resulting loading of the supporting structure, has to be assessed undeniably and can be estimated with this model. The input of the expected wind speed for the area, temperature and altitude, enable to optimise this wind load analysis.
In the case of palms, current breakage calculations, as employed for trees, cannot be employed yet for assessing the stem its safety. Hence, it seems that the only existing guideline is to combine the visual assessment of mechanical catastrophic behaviours with the calculation of the safety factor regarding elastic stability and the wind load analysis for cabling the palm tree when risk is suspected.

It is acknowledged that there is no statistical evidence yet to show a comparison between predicted results from the model against real outcomes, hence further research is needed in this field.

5. Consulted literature
Greenhill, G. 1881. Determination of the greatest height consistent with stability that a vertical pole or mast can be made, and the greatest height to which a tree of given proportions can grow. Proceedings of the Cambridge Philosophical Society 4: 65–73.
Sterken, P. 2006. Prognosis of the development of decay and the fracture safety of hollow
Hill.

6. Disclaimer:

The author, Peter Sterken, has made his best efforts to produce this informative and helpful
report. He has verified the technical accuracy of the information and contents of this report.
However, this information is complex and should be read with great care. Any persons
following the guidance in this publication should do so according to their own judgement and
expertise. The information in this report cannot replace or substitute for the services of trained
professionals in any field, including, but not limited to biological, arboricultural, engineering,
or legal fields. They don’t offer any professional, personal, legal or financial advice and none of
the information contained in this report should be confused as such advice. Any information
pertaining to the data, dates, events and other details relating to the trees, person or persons
and to the companies have been verified to the best. However, they make no representation or
warranties of any kind with regard to the contents of this report and Peter Sterken will not
accept any liability of any kind for any errors, omissions, loss, damage, death or injury to
persons or property or consequential loss (including loss or damage resulting from
negligence), caused or alleged to be caused directly or indirectly from using the information
contained here in.

Copyright notice: © Peter Sterken 2008. All intellectual property rights contained or in relation
to this work belong to Peter Sterken.
No part of this work may be used, copied, subsumed or exploited by any other author or person
or be stored in a retrieval system, transmitted or reproduced in any way, including but not
limited to digital copying and printing in any form whatsoever worldwide without the prior
agreement and written permission of the author.

www.sterken.be